

Evidence of Quasi-linear Super-Structures in the Cosmic Microwave Background and Galaxy Distribution

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ABSTRACT

Recent measurements of hot and cold spots on the cosmic microwave background (CMB) sky suggest a presence of super-structures on ($> 100 h^{-1}\text{Mpc}$) scales. We develop a new formalism to estimate the expected amplitude of temperature fluctuations due to the integrated Sachs-Wolfe (ISW) effect from prominent quasi-linear structures. Applying the developed tools to the observed ISW signals from voids and clusters in catalogs of galaxies at redshifts $z < 1$, we find that they indeed imply a presence of quasi-linear super-structures with a comoving radius $100 \sim 300 h^{-1}\text{Mpc}$ and a density contrast $|\delta| \sim O(0.1)$. We also find that the observed ISW signals are at odd with the concordant Λ cold dark matter (CDM) model that predicts Gaussian primordial perturbations at $\gtrsim 3\sigma$ level. We confirm that the mean temperature around the CMB cold spot in the southern Galactic hemisphere filtered by a compensating top-hat filter deviates from a mean value at $\sim 3\sigma$ level, implying that a quasi-linear supervoid or an underdensity region surrounded by a massive wall may reside at low redshifts $z < 0.3$ and the actual angular size ($16^\circ - 17^\circ$) may be larger than the apparent size ($4^\circ - 10^\circ$) discussed in literature. Possible solutions are briefly discussed.

Subject headings: cosmic microwave background – cosmology – large scale structure of the universe

1. Introduction

Although the Λ cold dark matter (CDM) models have succeeded in explaining a number of observations, some problems remain unresolved. For example, origins of a possible break of statistical isotropy in the large-angle cosmic microwave background (CMB) anisotropy

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(Tegmark et al. 2003; Eriksen et al. 2004; Vielva et al. 2004) and a possible discrepancy between observed and theoretically predicted galaxy-CMB cross-correlation (Rassat et al. 2007; Ho et al. 2008) are still not understood well. These observational results imply that structures on scales larger than $\gtrsim 100$ Mpc (super-structures) in our local universe are more lumpy than expected (Afshordi et al. 2009).

As the origin of the large-angle CMB anomalies, many authors have considered a possibility that the CMB is affected by local inhomogeneities (Moffat 2005; Tomita 2005a,b; Cooray & Seto 2005; Rakic & Schwartz 2007). Inoue & Silk (2006, 2007) have shown that a particular configuration of compensated quasi-linear supervoids can explain most of the features of the anomalies. Subsequent theoretical analyses have shown that the CMB temperature distribution for quasi-linear structures can be skewed toward low temperature due to the second order integrated Sachs-Wolfe (ISW)(or Rees-Sciama) effect (Tomita & Inoue 2008; Sakai & Inoue 2008).

In fact, Granett et al. (2008) found a significant ISW signal at the scale of $4^\circ - 6^\circ$ at redshifts around $z \sim 0.5$ and a weak signal of negatively skewed temperature distribution for distinct voids and clusters at redshifts $0.4 < z < 0.75$. Moreover, Francis & Peacock (2009) have shown that the ISW effect due to local structures at redshift $z < 0.3$ significantly affects the large-angle CMB anisotropies and that some of the CMB anomalies no longer persist after subtraction of the ISW contribution. These observations of galaxy-CMB cross-correlation may suggest an existence of anomalously large perturbations or new physics on scales > 100 Mpc.

In order to evaluate the significance of the ISW signals for prominent structures, N-body simulations on cosmological scales seem to be suitable(Cai et al. 2010) for this purpose. However, the computation time is relatively long and finding physical interpretation from a number of numerical results is sometimes difficult. In contrast, analytical methods are suitable for estimating the order of statistical significance in a relatively short time, and physical interpretations are often simpler.

In this paper, we evaluate the statistical significance of the ISW signals for prominent super-structures based on an analytic method and try to construct simple models that are consistent with the data. In section 2, we develop a formalism for analytically calculating the ISW signal due to prominent non-linear super-structures based on a spherically symmetric homogeneous collapse model and we study the effect of non-linearity and inhomogeneity of such structures. In section 3, we apply the developed method to observed data and calculate the statistical significance of the discrepancy between the predicted and the observed ISW signals. In section 4, we discuss the origin of the observed discrepancy. In section 5, we summarize our results and discuss some unresolved issues. In the following,

unless noted, we assume a concordant Λ CDM cosmology with $(\Omega_{m,0}, \Omega_{\Lambda,0}, \Omega_{b,0}, h, \sigma_8, n) = (0.26, 0.74, 0.044, 0.72, 0.80, 0.96)$, which agrees with the recent CMB and large-scale structure data (Sánchez et al. 2009).

2. Cross correlation for prominent quasi-linear structures

2.1. Thin-shell approximation

For simplicity, in this section, we assume that super-structures are modelled by spherically symmetric homogeneous compensated voids/clusters with an infinitesimally thin-shell. The background spacetime is assumed to be a flat FRW universe with matter and a cosmological constant Λ .

Let κ and ξ be the curvature and the physical radius of a void/cluster in unit of the Hubble radius H^{-1} , and δ_H as the Hubble parameter contrast, t denotes the cosmological time. We describe the angle between the normal vector of the shell and the three dimensional momentum of the CMB photon that leaves the shell by ψ . We assume that the comoving radius r_v of the void in the background coordinates satisfies $r_v \propto t^\beta$, where β is a constant.

Up to order $O((r_v/H^{-1})^3)$ and $O(\kappa^2)$, the temperature anisotropy of the CMB photons that pass through spherical homogeneous compensated voids in the flat FRW universe can be written as (Inoue & Silk 2007),

$$\begin{aligned} \frac{\Delta T}{T} &= \frac{1}{3} \left[\xi^3 \cos \psi \left(-2\delta_H^2 - \delta_H^3 + (3 + 4\delta_m)\delta_H\Omega_m \right. \right. \\ &\quad + \delta_m\Omega_m(-6\beta/s + 1) + (2\delta_H^2 + \delta_H^3 + \delta_m\Omega_m \\ &\quad \left. \left. + (3 + 2\delta_m)\delta_H\Omega_m) \cos 2\psi \right) \right], \end{aligned} \quad (1)$$

$$t = sH^{-1}, \quad s = \frac{2}{3\sqrt{1-\Omega_m}} \ln \left[\frac{\sqrt{1-\Omega_m} + 1}{\sqrt{\Omega_m}} \right], \quad (2)$$

where δ_m and Ω_m denote the matter density contrast of the void and the matter density parameter, respectively. The variables $\xi, \psi, \delta_m, \delta_H$, and Ω_m are evaluated at the time the CMB photon leaves the shell. It should be noted that the formula (1) is valid even if $|\delta_m|$ or $|\delta_H|$ is somewhat large as long as the normalized curvature κ is sufficiently small. The formula (1) can be also applied to spherical compensating clusters with a density contrast $\delta_m > 0$ corresponding to a homogeneous spherical cluster with an infinitesimally thin “wall”. This approximation holds only in weakly non-linear regime since the amplitude of the density contrast corresponding to a negative mass cannot exceed 1. We examine this approximation in sections 2-4 in detail.

Because we are mainly interested in linear $|\delta_m| \ll 1$ and quasi-linear $|\delta_m| = O(0.1)$ regime, we expand δ_H in terms of δ_m up to second order as

$$\delta_H = \Omega_m \delta_m (1 + 1/f(w))/2 - \epsilon \delta_m^2, \quad (3)$$

where w is an equation-of-state parameter, ϵ is a constant that describes the non-linear effect and

$$f(w) = -\frac{3}{5}(1+w)^{1/3} {}_2F_1\left[\frac{5}{6}, \frac{1}{3}, \frac{11}{6}, -w\right], \quad (4)$$

where ${}_2F_1$ is Gauss' hypergeometric function. ϵ can be estimated from numerical integration of the Friedmann equation inside the shell as we shall show later. In a similar manner, for the shell expansion, we assume the following relation for the wall peculiar velocity normalized by the background Hubble expansion,

$$\tilde{v} = s^{-1}\beta, \quad \beta = -\frac{f(\Omega_m)}{6}\delta_m + \nu\delta_m^2, \quad (5)$$

where ν represents a constant that describes the non-linear effect (Inoue & Silk 2007). $f(\Omega_m)$ is written in terms of the scale factor a and the growth factor D as

$$f(\Omega_m) = \frac{a}{D} \frac{dD}{da} \sim \Omega_m^{0.6}. \quad (6)$$

In quasi-linear regime, simplification $\epsilon = \eta = 0$ can be verified, which will be shown in section 2-3 and section 2-4.

2.2. Homogeneous collapse

In order to describe the dynamics of local inhomogeneity, we adopt a homogeneous collapse model which consists of an inner FRW patch and a surrounding background flat FRW spacetime (Lahav et al. 1991). The size of the inner patch is assumed to be sufficiently smaller than the horizon H^{-1} in the background spacetime.

We assume that both the regions have only dust and a cosmological constant Λ . The time evolution of either the inner patch or the background spacetime is described by the Friedmann equation,

$$\frac{H^2}{H_0^2} = \frac{\Omega_{m,0}}{a^3} + \Omega_{\Lambda,0} + \frac{1 - \Omega_{tot}}{a^2}, \quad (7)$$

where a denotes the scale factor, $\Omega_{m,0}, \Omega_{\Lambda,0}, \Omega_{tot}$ are the present energy density parameters of non-relativistic matter, a cosmological constant Λ , and the total energy density, respectively. The scale factor at present for the background spacetime is set to $a_0 = 1$. In what follows, we describe variables in the inner patch by putting tilde “ \sim ” on top of the variables and we consider only flat FRW universes with dusts and a cosmological constant Λ .

First, we calculate matter density contrast δ_m of the inner patch. Initially ($z_i \gg 1$), we assume that the fluctuation of the matter perturbation δ_{mi} is so small that $\tilde{a}_i \approx a_i, \tilde{H}_i \approx H_i$.

Then, the Friedmann equation (7) yields,

$$\frac{\tilde{H}^2}{H_0^2} = \frac{\Omega_{m,0}(1 + \delta_{mi})}{\tilde{a}^3} + \Omega_{\Lambda,0} - \frac{\delta_{mi}\Omega_{m,0}}{\tilde{a}^2\tilde{a}_i}. \quad (8)$$

In terms of physical radius of the patch $\tilde{R} = \tilde{a}r$, where r is the comoving radius measured in the background spacetime, equation (8) can be written as

$$\begin{aligned} \left(\frac{d\tilde{R}}{dt}\right)^2 &= H_0^2 \left[-\Omega_{m,0}(1 + z_i)^3 \tilde{R}_i^2 \delta_{mi} + \Omega_{m,0}(1 + z_i)^3 \right. \\ &\quad \times \left. (1 + \delta_{mi}) \frac{\tilde{R}_i^3}{\tilde{R}} + \Omega_{\Lambda,0} \tilde{R}^2 \right], \end{aligned} \quad (9)$$

where t is the cosmological time. The matter density contrast δ_m can be written as a function of a ratio of the present and the initial comoving radius $\eta \equiv r/r_i$ as $\delta_m = \eta^{-3} - 1$. From equation (9), η as a function of redshift z is given by solving

$$\begin{aligned} \left(\frac{d\eta}{dz}\right) &= - \left[-\Omega_{m,0}(1 + z_i)\delta_{mi} + \Omega_{m,0}(\eta/(1 + z))^{-1} \right. \\ &\quad + \left. \Omega_{\Lambda,0}(\eta/(1 + z))^2 \right]^{1/2} [\Omega_{m,0}(1 + z)^3 + \Omega_{\Lambda,0}]^{-1/2} \\ &\quad + \frac{\eta}{1 + z}. \end{aligned} \quad (10)$$

Note that right-hand-side in equation (10) does not depend on \tilde{R}_i . From numerical integration of equation(10), the matter density contrast δ_m as a function of redshift z is obtained by setting initial density contrast $\delta_{mi} = \delta_m(z_i)$.

The linearly perturbed matter density contrast δ_m^L in the FRW background spacetime is given by

$$\delta_m^L(z) = \frac{3\delta_{mi}H(z)}{5} \int_z^\infty du \frac{u+1}{H^3(1/u-1)}, \quad (11)$$

where $H^2(z) = \Omega_{m,0}(1 + z)^3 + \Omega_{\Lambda,0}$ (Heath 1977). Constant 3/5 comes from our choice of initial condition that the peculiar velocity inside the initial patch is zero. The relation between δ_m and δ_m^L is shown in figure 1. Because the relation does not change so much even if one varies the cosmological parameters of the background spacetime, non-linear isolated homogeneous spherical patches can be solely calculated from corresponding linear perturbations (Friedmann & Piran 2001). It should be noted, however, that the relation is valid only if $\delta_m \gtrsim \delta_{v,cut} = -0.8$ because of shell-crossing (Furlanetto & Piran 2006).

Next, we calculate the Hubble parameter contrast $\delta_H = \tilde{H}/H - 1$ in non-linear regime.

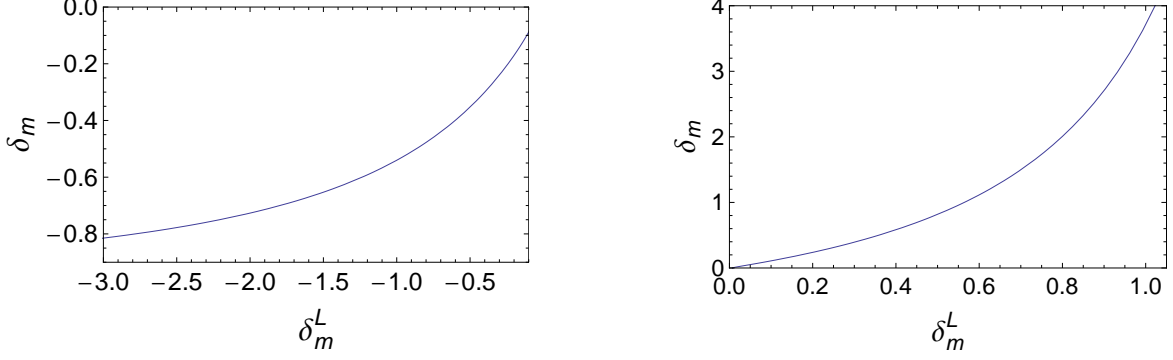


Fig. 1.— Density contrast δ_m as a function of linear density contrast δ_m^L for voids (left) and clusters (right).

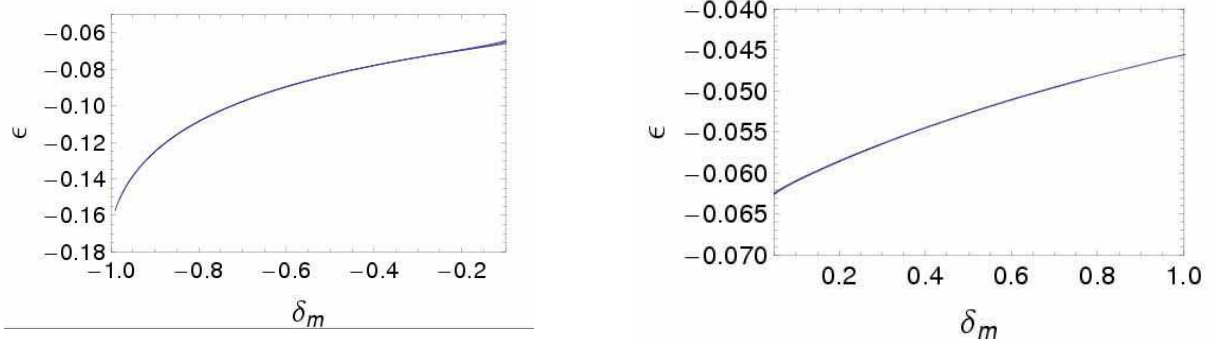


Fig. 2.— Non-linear parameter ϵ as a function of density contrast δ_m in the EdS model.

Plugging $\eta = R(1+z)/(R_i(1+z_i))$ into equation (8), we have

$$\begin{aligned}
 & \tilde{H}^2(\Omega_{m,0}(1+z)^3 + \Omega_{\Lambda,0}) \\
 &= H^2(\Omega_{m,0}(1+\delta_{mi})(1+z)^3\eta^{-3} + \Omega_{\Lambda,0} \\
 &- \delta_{mi}\Omega_{m,0}(1+z_i)(1+z)^2\eta^{-2}),
 \end{aligned} \tag{12}$$

where η is given by solving equation (10). From equation (3) and (12), one can estimate the non-linear parameter as

$$\epsilon = \delta_m^{-1}\Omega_{m,0}(1+1/f(w))/2 - \delta_m^{-1}(\tilde{H}/H - 1). \tag{13}$$

2.3. Effect of non-linear dynamics

In previous section, we have seen that the non-linear density contrast δ_m for a spherically symmetric homogeneous patch can be written in terms of corresponding linear density

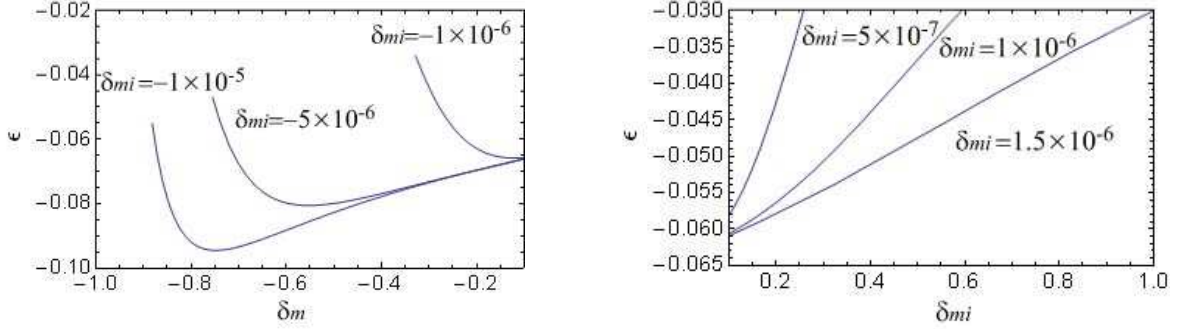


Fig. 3.— Non-linear parameter ϵ as a function of density contrast δ_m in the flat- Λ model with $\Omega_{m,0} = 0.26$.

contrast δ_m^L . In order to calculate the ISW effect, we need to estimate the Hubble parameter contrast δ_H and the peculiar velocity \tilde{v} of the wall. The non-linear corrections to δ_H and \tilde{v} can be characterized by two parameters ϵ and ν , respectively.

First, we consider the effect of non-linear correction to the Hubble contrast δ_H . As one can see in figure 2 and 3, ϵ is always negative and the amplitude is $|\epsilon| < 0.16$ for $|\delta_m| < 1.0$. This represents a slight enhancement in the expansion speed within the inner patch due to non-linearity. In low-density universes ($\Omega_{m,0} < 1$), $|\epsilon|$ is smaller than that in high-density universes. In the Einstein-de Sitter (EdS) universe, ϵ depends only on δ_m (figure 2). In contrast, in low-density universes, ϵ depends on the amplitude of the initial epoch as well (figure 3). This is because the expansion speed inside the patch is suppressed when the energy component of the background universe is dominated by a cosmological constant Λ . We have found that the non-linear contribution to δ_H is less than 10 per cent for $|\delta_m| < 0.2$ and $\Omega_m > 0.26$.

Second, we consider the effect of non-linear correction to peculiar velocity of the wall. In the thin-shell limit, the motion of the spherically symmetric wall can be obtained by numerically solving a set of ordinary differential equations using Israel's method (Israel 1966, Maeda & Sato 1983). If the inner region and the outer region are described by the FLRW spacetime, the fitting formula for the peculiar velocity of an expanding wall normalized by the background Hubble expansion can be written as (Maeda, Sakai, & Triay 2010)

$$\tilde{v} = \frac{\Omega_m^{0.56}}{6} (|\delta_m| + 0.1\delta_m^2 + 0.07|\delta_m|^3), \quad (14)$$

for $\Omega_m + \Omega_\Lambda = 1$. We have confirmed that the accuracy of the fitting formula is within one percent for $0 < \Omega_m \leq 1$ and $|\delta_m| < 1$ using numerically computed values. From equation (14), we find that the contribution of non-linear effect is less than 5 percent for $|\delta_m| < 0.3$. Thus an approximation $\delta_H \propto \delta_m \propto \tilde{v}$ can be validated in the quasi-linear regime.

2.4. Effect of non-linearity and inhomogeneity on the ISW signal

In literature, the thin-shell approximation has been often used to describe almost empty voids with $\delta \sim -1$ (Maeda & Sato 1983). In quasi-linear regime, however, we also need to consider the effect of thickness of the wall and inhomogeneity of the matter distribution because quasi-linear voids are not in the asymptotic regime. Non-linearity of the wall may significantly affect the CMB photons that pass through it. Moreover, it seems not realistic to apply the thin-shell approximation to spherically symmetric clusters since the mass of the wall cannot be negative.

In order to estimate the validity of the thin-shell approximation, we have compared the ISW signal with those obtained by using second order perturbation theory (Tomita & Inoue 2008) and by using the Lemaitre-Tolman-Bondi (LTB) solution (Sakai & Inoue 2008), which yields exact results without recourse to the cosmological Newtonian approximation. We have assumed top-hat type matter distribution (for linear matter perturbation) for void/cluster for calculation using second order perturbation theory and a smooth distribution specified by a certain polynomial function for calculation using the LTB solution. The voids/clusters are assumed to be compensated so that the gravitational potential outside the cut-off radius (r_1 for the perturbative analysis and r_{out} for the LTB-based analysis) is constant. For detail, see appendices A and B.

As an example, we have computed temperature fluctuations ΔT generated from a compensated void/cluster using the three types of method. The density contrast, the comoving radius and the redshift of the center of a void/cluster are set to $|\delta_m| = 0.1$, $r_v = 200 h^{-1}\text{Mpc}$, $r_1 = r_{out} = 210 h^{-1}\text{Mpc}$, and $z = 0.2$, respectively. The width of the wall is assumed to be 1/10 of the cut-off radius. As one can see in figure 4, the three methods agree well for low density universes in which the linear effect is dominant. In contrast, the discrepancy becomes apparent for high density universes in which the non-linear effect is dominant. This discrepancy is partially due to a slight difference in the assumed density profile (top-hat type for the perturbative analysis, polynomial type for the LTB). In order to demonstrate the role of non-linearity, we have plotted first order (linear ISW effect) and second order (RS effect minus linear ISW effect) contributions to the ISW signal (figure 5). The first order effect makes the CMB temperature negative(positive) for a void(cluster) but the second order effect makes the CMB temperature negative near the center and positive near the boundary regardless of the sign of the density contrast. As a result, the amplitude of temperature fluctuation for a void(cluster) is enhanced(suppressed) in the direction near the center but it is suppressed(enhanced) in the direction near the boundary. These non-linear effects become much apparent for models with higher background density because the linear ISW effect becomes less effective. However, if we take into account of the thickness of the

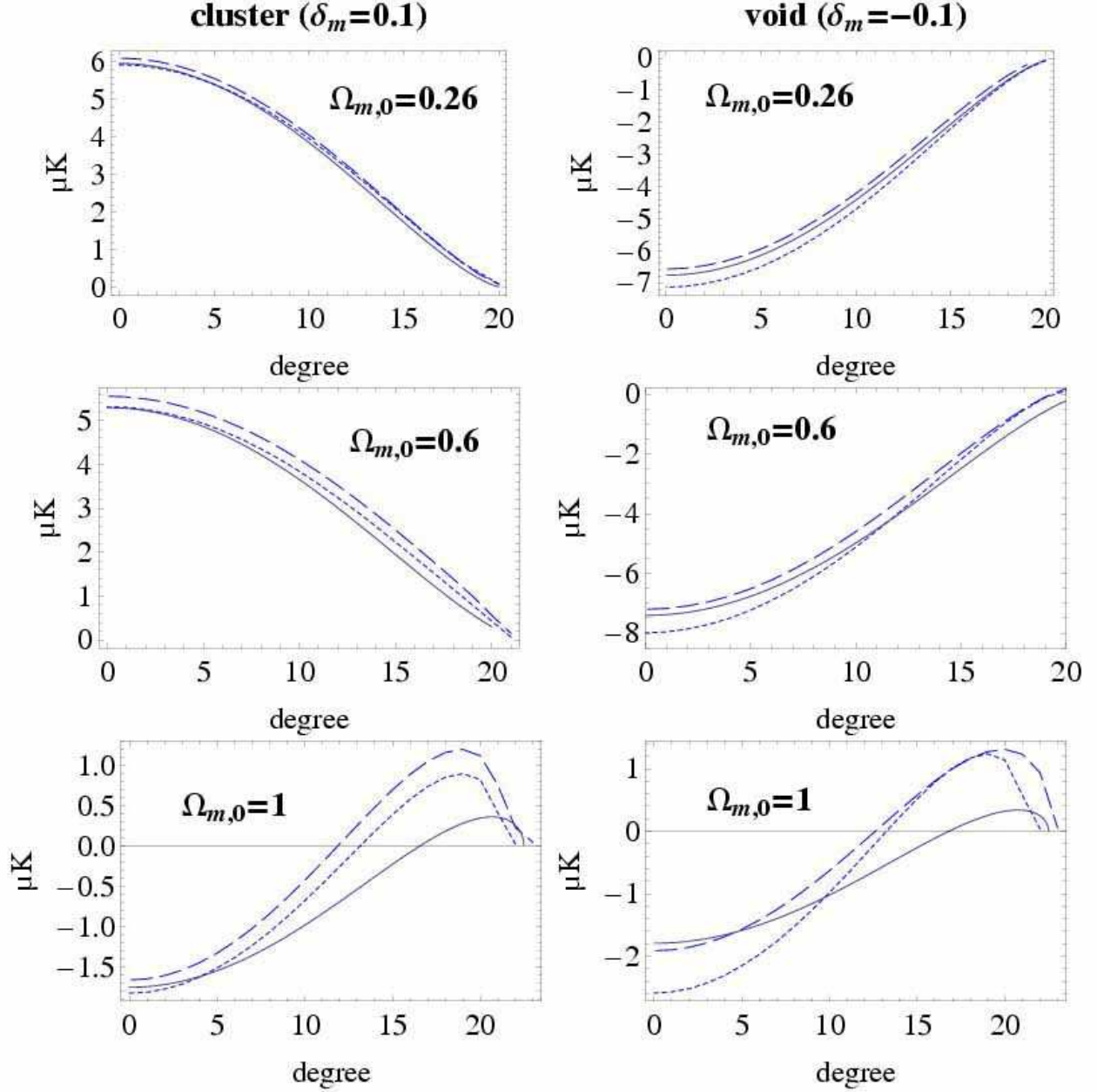


Fig. 4.— Temperature fluctuation ΔT due to a compensated cluster (left panels)/a compensated void (right panels) centered at redshift $z = 0.2$ as a function of angular radius from the center. We used three methods for deriving the ISW signal: the thin-shell approximation (thick curve), the second order perturbation theory (dotted curve) and the LTB solution (dashed curve). We set the comoving (outer) radius of the cluster/void $r_v = 200 h^{-1} \text{Mpc}$ for the thin-shell approximation, $r_{out} = 210 h^{-1} \text{Mpc}$ for the second order and the LTB calculations with wall width about tenth of the outer radius. The density parameter is $\Omega_{m0} = 0.26$. For detail, see the text.

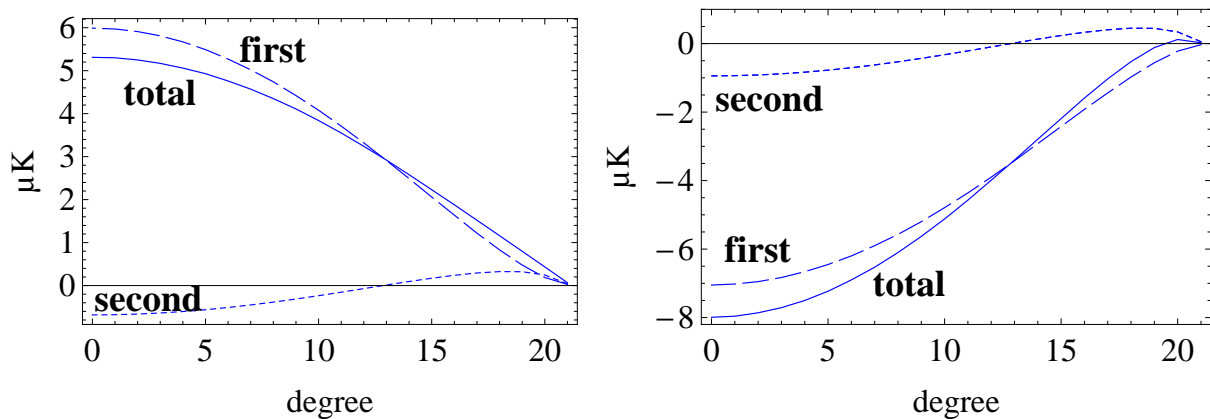


Fig. 5.— The first and the second order contributions to temperature fluctuation ΔT due to a compensating cluster (left)/void (right) centered at redshift $z = 0.2$ as a function of an angular radius from the center. We set the outer comoving radius $r_{out} = 210 h^{-1}\text{Mpc}$, the inner comoving radius $r_{in} = 0.93 r_{out}$, the density parameter $\Omega_{m0} = 0.6$, and the density contrast at the center of the cluster/void $|\delta_m(z = 0.2)| = 0.1$.

wall, these non-linear effects can be less conspicuous since the amplitude of the gravitational potential becomes smaller for a fixed outer radius (figure 6). In the Λ -dominated universe, a quasi-linear compensated void can be recognized as a cold spot surrounded by a very weak hot ring, whereas a quasi-linear compensated cluster can be recognized as a hot spot possibly with a dip at the center of it. In the EdS universe, either a compensated quasi-linear void or a cluster can be identified as a cold spot surrounded by a hot ring.

2.5. ISW effect from prominent quasi-linear structures

In order to fully utilize information of the three dimensional distribution, we consider a temperature anisotropy $\Delta T/T$ obtained from stacked images on the CMB sky that corresponds to most prominent voids/clusters in a galaxy catalog. First, we fix an angular radius θ_{out} of a circular region on the CMB sky that will be used in the stacking analysis. Then, the corresponding smoothing scale r_s in comoving coordinate for the corresponding fluctuation at z is $r_s = (1+z)D_A(z)\theta_{out}$, where $D_A(z)$ is the angular diameter distance to the galaxy. The corresponding initial smoothing scale is $r_s^i = \eta^{-1}r_s$.

We assume that the probability distribution function (PDF) of linear density contrast

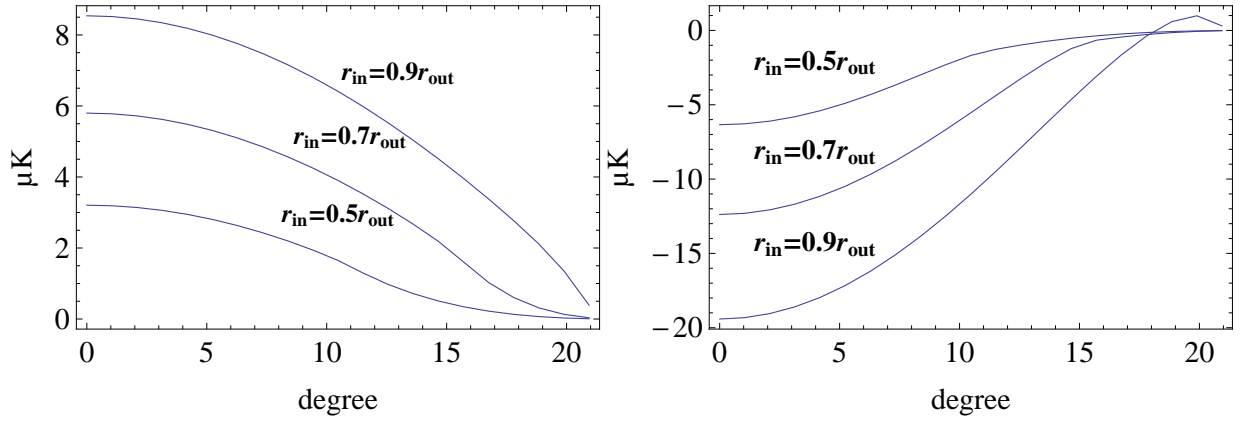


Fig. 6.— The effect of thickness of the wall for $\Omega_{m0} = 0.6$ $r_1 = 210 h^{-1}\text{Mpc}$, and $\delta_m(z = 0.2) = 0.2$ (left) and $\delta_m(z = 0.2) = -0.2$ (right) for top-hat type density perturbations (see appendix A).

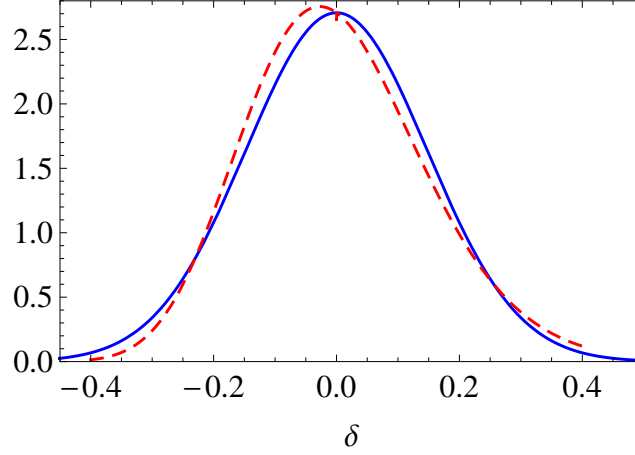


Fig. 7.— Effect of non-linearity: the PDF of δ_m (dashed curve) and that of δ_m^L (full curve) for a smoothing scale $r_s = 50h^{-1}\text{Mpc}$ at $z = 0.5$.

δ_m^L at redshift z is given by a Gaussian distribution function,

$$P^L(\delta_m^L; \sigma(r_s^i, z)) = \frac{1}{\sqrt{2\pi}\sigma(r_s^i, z)} \exp\left[-\frac{(\delta_m^L)^2}{2\sigma^2(r_s^i, z)}\right] \quad (15)$$

where $\sigma^2(r_s^i, z)$ is the variance of the linearly extrapolated density contrast at redshift z smoothed by a spherically symmetric top-hat type window function with an initial comoving radius r_s^i . Note that $\sigma(r_s^i, z)$ depends on cosmological parameters such as $\Omega_{m,0}$, σ_8 and the spectrum index n . Then, the PDF of non-linear density contrast of the inner patch δ_m is given by

$$P^{NL}(\delta_m; \sigma(r_s^i, z)) = \alpha P^L(\delta_m^L(\delta_m); \sigma(r_s^i, z)) \frac{d\delta_m^L}{d\delta_m}, \quad (16)$$

where α is a constant that normalizes the PDF.

As shown in figure 7, the PDF of δ_m is positively skewed in comparison with the PDF of δ_m^L because of non-linearity. For a sample region at redshift z with a total comoving volume V , the total number of voids or clusters with a radius r_s is approximately $N_t \approx 3V/(4\pi r_s^3)$. In what follows we assume that the number of prominent voids/clusters (N_v/N_c) determines the corresponding threshold of density contrast $\delta_{m,th}(z)$, which is given by

$$N_v/N_t = \int_{\delta_{v,cut}}^{\delta_{m,th}} P^{NL}(\delta_m; \sigma(r_s^i, z)) d\delta_m, \quad (17)$$

and

$$N_c/N_t = \int_{\delta_{m,th}}^{\delta_{c,cut}} P^{NL}(\delta_m; \sigma(r_s^i, z)) d\delta_m, \quad (18)$$

respectively. From equations (1), (17), and (18) the mean temperature fluctuation within an angular radius θ_{out} for a stacked N_v or N_c images corresponding to prominent quasi-linear voids/clusters at redshift $\sim z$ can be approximately written as

$$\langle \Delta T \rangle = \frac{\iint W(\theta; \theta_{in}) \Delta T(\delta_m, \psi(\theta)) P^{NL}(\delta_m; \sigma) d^2 \theta d\delta_m}{\pi \theta_{th}^2 \int P^{NL}(\delta_m; \sigma) d\delta_m}, \quad (19)$$

where $0 \leq \theta \leq \theta_{out}$, $W(\theta; \theta_{in})$ is a compensating window function that satisfies

$$\int_0^{\theta_{in}} 2\pi\theta W(\theta; \theta_{in}) d\theta = - \int_{\theta_{in}}^{\theta_{out}} 2\pi\theta W(\theta; \theta_{in}) d\theta \quad (20)$$

and

$$\psi(\theta) = \theta + \arcsin \left[h^{-1} \cos \theta \sin \theta - \sqrt{\sin^2 \theta - h^{-2} \sin^4 \theta} \right], \quad (21)$$

where $h = r_s(1+z)/D_A(z)$.

3. Application to observations

3.1. SDSS-WMAP cross correlation

A cross correlation analysis using a stacked image built by averaging the CMB surrounding distinct voids/clusters has been done by Granett et al. (2008). They have used 1.1 million Luminous Red Galaxies (LRGs) from the SDSS catalog covering 7500 square degrees. The range of redshifts of the LRGs is $0.4 < z < 0.75$, with a median of ~ 0.5 . The total volume is $\sim 5 h^{-3} \text{Gpc}^3$. They used so-called the ZOBOV (ZOnes Bordering On Voidness; Neyrinck 2008) algorithm to find supervoids and superclusters in the LRG catalog and made a stacked image from an inversely variance weighted WMAP 5-year (Q,V, and W) map. In order to reduce contribution from CMB fluctuations on scales larger than the objects, they used a top-hat type compensating filter

$$W_{th}(\theta; \theta_{in}) = \begin{cases} 1 & (\theta < \theta_{in}) \\ -1 & (\theta_{in} \leq \theta \leq \theta_{out}), \end{cases} \quad (22)$$

where $\theta_{out} = \cos^{-1}(2 \cos \theta_{in} - 1)$.

First, using the developed tools based on thin-shell approximation and homogeneous collapse model in section 2, we estimate the expected amplitude of the ISW signal for prominent structures in a concordant Λ CDM model with Gaussian primordial fluctuations and compare with the observed values obtained from the SDSS-LRG catalog. The number of

most distinct voids or clusters N and the cut-off radius θ_{in} are chosen as free parameters. At redshift $z = 0.5$, the mean density contrast filtered by a top-hat type function with radius $r = 130 h^{-1}\text{Mpc}$ corresponding to $\theta_{out} \sim 5.6^\circ$ is just $\langle \delta_m^L \rangle = 0.046$ and the background density parameter is $\Omega_m(z = 0.5) = 0.54$. Because the influence of non-linear ISW effect is weaker than that of the linear ISW effect in this setting, we expect that details of non-linear calculations will not much affect the result. In what follows, we use an approximation $\delta_H \propto \delta_m \propto \tilde{v}$, where δ_m is determined from the homogeneous collapse model in section 2.

As shown in table 1, it turned out that the expected values of the ISW(Rees-Sciama) signal are typically of the order of $O(10^{-7})\text{K}$. As expected, the amplitude gets larger as the number of stacked image decreases, and the amplitude for voids systematically becomes larger than those for clusters by 5 – 10 percent(Tomita & Inoue 2008; Sakai & Inoue 2008). On the other hand, the order of the observed amplitudes are extremely large as $O(10^{-6})\text{K}$. It turns out that the discrepancy remains at $3 - 4\sigma$ level for $N = 30$ and $N = 50$.

Second, we reconstruct the mean density profile from the observed ISW signal for the SDSS-LRG catalog using our LTB model. From figure 7, one can notice a hot ring around a cold spot for the stacked image of voids and a dip at the center of a hot spot for the stacked image of clusters. Although the amplitude of the hot-ring cannot be reproduced well, the observed dip at the center of the hot spot can be qualitatively reproduced in our LTB models. We have found that the dip at the center of a compensated cluster can be generated only if the linear ISW effect balances the non-linear ISW effect in a limited parameter region. Thus the observed features in stacked images strongly imply that the corresponding super-structures are not linear but quasi-linear or non-linear objects. The density fluctuations which are necessary to produce the observed ISW signals are found to be tremendously large. In figure 8, we plot the ISW signal from a compensated cluster with $\delta_m \sim 7\sigma$ and that from a compensated void with $|\delta_m| \sim 10\sigma$ at $z = 10$ (see the radial density profiles at figure A1 in appendix A). Even for these very rare objects, the amplitudes of ISW signals in our LTB models are much smaller than the observed ones. In fact, the mean temperatures for a compensating filter $\theta_{in} = 4^\circ$ are $3.6\mu\text{K}(1.4\sigma)$ and $-3.1\mu\text{K}(2.6\sigma)$ for the cluster and the void, respectively. On the other hand, the probability of generating these fluctuations is as extremely small as 10^{-12} in standard inflationary models that predict primordial gaussianity.

Thus, the observed large ISW signals for the stacked image strongly suggest a presence of super-structures on scales $O(100 h^{-1}\text{Mpc})$ with anomalously large density contrast $O(0.1)$ which can not be produced in the concordant LCDM model.

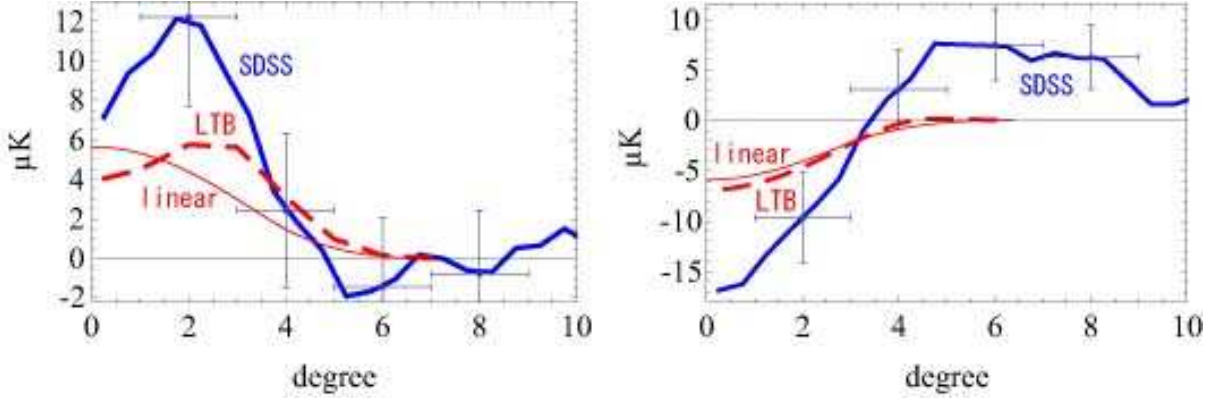


Fig. 8.— Observed mean CMB temperature radial profiles for a stack image of 50 clusters (left) and that of 50 voids (right) (thick full curves) in the SDSS-LRG catalog (Granett et al 2008) and the corresponding theoretical radial profiles for compensated spherical clusters (left)/void (right) based on the LTB solution (dashed curves) and the linear calculation (thin full curves) with a top-hat type compensated density distribution corresponding to the LTB solutions. The LTB parameters for the set of clusters are $(r_{out}, r_{in}, \delta_m, z_c) = (0.055H_0^{-1}, 0.022H_0^{-1}, 1.0, 0.5)$ and those for the set of voids are $(0.050H_0^{-1}, 0.015H_0^{-1}, -0.55, 0.5)$. The effective radii of the inner patch at a redshift $z = 10$ and the mean filtered temperature with $\theta_{in} = 4^\circ$ are $(0.029H_0^{-1}, 3.6\mu\text{K})$ (cluster) and $(0.023H_0^{-1}, -3.1\mu\text{K})$ (void), respectively. Evolution of the density profile in the LTB models are shown in appendix B. 1σ error bars are obtained from 1000 Monte Carlo simulations on the WMAP7 Q+V+W map smoothed at 1° scale (see section 3.3).

3.2. 2MASS-WMAP cross correlation

Francis & Peacock (2009) estimated the local density field in redshift shells using photometric redshifts for the 2MASS galaxy catalog. They reconstructed the CMB anisotropies due to the ISW effect from the local density field δ_m . They approximated the bias in each redshift shell by a linear bias relation $\delta_g = b \delta_m$ and assumed that the bias is independent of scale and redshift in each shell. In order to obtain the bias parameter b , a maximum likelihood analysis of the galaxy catalog was performed.

There are two prominent spots in the reconstructed CMB anisotropy. One is a hot spot due to a supercluster around the Shapley concentration at redshifts $0.1 \leq z \leq 0.2$. Another one is a cold spot due to a supervoid at redshifts $0.2 \leq z \leq 0.3$ in the direction to $(l, b) \sim (0, -30^\circ)$. The angular radii of both structures are $\theta = 20^\circ - 30^\circ$. The temperatures near the center of both structures are $\sim 20 \mu\text{K}$. The position of the supervoid is very close

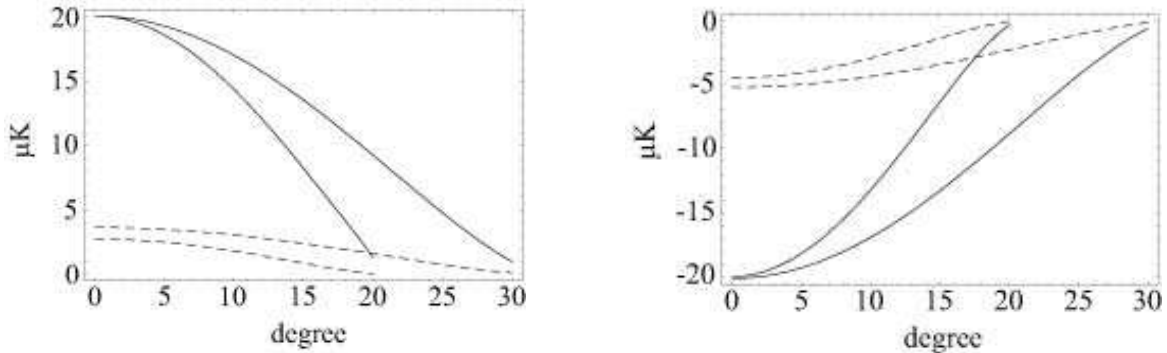


Fig. 9.— Left: Theoretically modeled temperature profiles for an observed (full curves) and an expected (dotted curves) supercluster in the 2MASS catalog. Right: modeled temperature profiles for an observed (full curves) and an expected supervoid (dotted curves). These superstructures reside at redshifts $z = 0.1 - 0.3$. We have plotted two possible profiles for each structure since there is an ambiguity in the angular size due to errors ($\Delta z \sim 0.03$) in photometric redshifts in the 2MASS galaxy catalog (Francis & Peacock 2009).

to the one predicted in Inoue & Silk (2007), $(l, b) \sim (330^\circ, -30^\circ)$.

Based on developed methods in section 2, we have estimated the expected density contrast δ_m and the corresponding temperature profile (figure 9) due to a most prominent object in the shell. We have assumed the same cosmological parameters and primordial gaussianity as those discussed in section 3. In order to compute the temperature profile, we have used a homogeneous thin-shell model. As shown in table 2, the observed density contrasts are larger by 4-7 times the expected values. If the comoving radius of the structure is $\sim 200 h^{-1}\text{Mpc}$, the absolute value of the density contrast should be $|\delta_m| = 0.2 - 0.3$, which implies a presence of anomalous quasi-linear super-structures. Our result is consistent with the power spectrum analysis in Francis & Peacock (2009) where a noticeable excess of the observed power at low multipoles $2 \leq l \leq 4$ was reported.

3.3. The CMB cold spot

The most striking CMB anomaly is the presence of an apparent cold spot in the Wilkinson Microwave Anisotropy Probe (WMAP) data in the Galactic southern hemisphere (Vielva et al. 2004; Cruz et al. 2005) (see figure 10). The cold spot has a less than 2 per cent probability of being generated as random gaussian fluctuations (Cruz et al. 2007a), if one uses spherical mexican-hat type wavelets as filter functions (see also Zhang & Huterer 2010).

Table 1. Expected and observed amplitude of mean temperature for a compensating filter
 $\theta_{in} = 4^\circ$

N	cluster (μK)	void (μK)	average(μK)
1	0.98	-1.2	1.07
5	0.82	-0.94	0.88
10	0.73	-0.83	0.78
30	0.57	-0.64	0.61 (11.1 \pm 2.8) ^a
50	0.48 (7.9 \pm 3.1) ^a	-0.53 (-11.3 \pm 3.1) ^a	0.51 (9.6 \pm 2.2) ^a
70	0.42	-0.46	0.42 (5.4 \pm 1.9) ^a

^aTaken from Granett et al. (2008).

Table 2. Expected and observed density contrast for super-structures in 2MASS galaxy catalog

radius ^a	expected	observed	radius ^a	expected	observed
230	0.037	0.20	370	-0.013	-0.049
150	0.094	0.69	250	-0.037	-0.15

^aThe unit of the radii is h^{-1} Mpc.

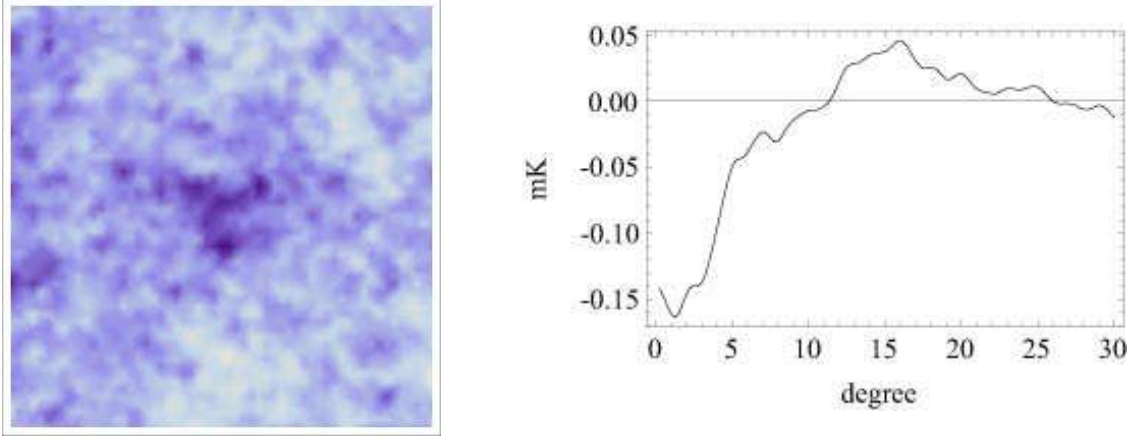


Fig. 10.— Left: the WMAP7 ILC temperature map ($40^\circ \times 40^\circ$) smoothed at 1° scale. Right: the averaged radial profile of the ILC map as a function of inclination angle θ from the center of the cold spot $(l, b) = (207.8^\circ, -56.3^\circ)$. A peak at $\theta \sim 15^\circ$ corresponds to a hot ring.

Assuming that it is not a statistical artifact, a variety of theoretical explanations have been proposed, such as galactic foreground (Cruz et al. 2006), texture (Cruz et al. 2007b), and Sunyaev-Zeldovich (SZ) effect. However, these models failed to explain other large-angle anomalies by the same mechanism.

Inoue & Silk (2006, 2007) proposed that the cold spot may be produced by a supervoid at $z < 1$ in the line-of-sight due to the ISW effect and have shown that another pair of supervoids that are tangential to the Shapley concentration can explain the alignment between the quadrupole and the octopole in the CMB. Subsequently, Rudnick et al. (2007) found a depression in source counts in the NRAO VLA Sky Survey (NVSS) in the direction to the cold spot, although the statistical significance has been questioned (Smith & Huterer 2010). Recent optical observations (Granett et al. 2009, Bremer et al. 2010), however, revealed that any noticeable supervoids at $0.35 < z < 1$ in the line-of-sight are ruled out. These observations suggest that the angular size of the supervoid may be larger or smaller than expected and that it resides at low redshifts $z < 0.35$ or at high redshifts $z > 1$.

In order to test such a possibility, we have calculated averaged temperature around the cold spot (see figure 11) using a spherical top-hat compensating filter $W_{th}(\theta; \theta_{in})$.

Interestingly, we have discovered two peaks in the plot of the filtered mean temperature around the cold spot as a function of inner radius of the filter (figure 12). The inner and the outer peaks are observed at $\theta_{in} = 4^\circ - 5^\circ$ ($\theta_{out} = 6^\circ - 7^\circ$) and $\theta_{in} = 12^\circ - 13^\circ$ ($\theta_{out} = 16^\circ - 17^\circ$). The outer peak corresponds to a hot ring, which is visible by eyes (see figure 10).

In order to estimate the statistical significance of the peaks, we have used a WMAP

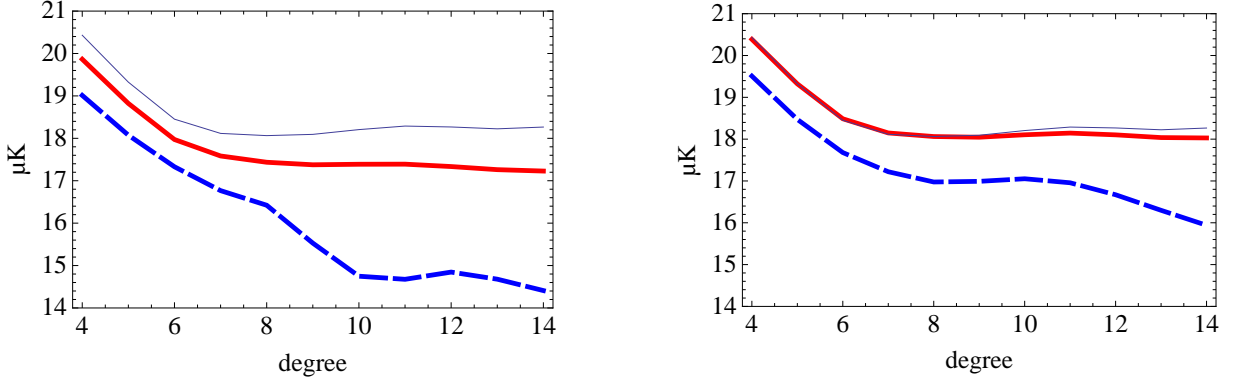


Fig. 11.— Left: 1σ of the filtered mean temperature as a function of an inner radius θ_{in} for the Q+V+W map (dashed curve) and that derived from C_l 's (thin curve) and from pseudo- C_l 's (thick curve). Right: 1σ for the ILC map (dashed curve) and corresponding theoretical values (thin curve and thick curve) as shown in the left figure.

7-year internal linear combination (ILC) map smoothed at 1° scale with a Galactic skycut $|b| < 20^\circ$ and a combination of the Q, V, and W band frequency maps smoothed at 1° scale averaged with weights inversely proportional to the noise variances with a “standard” Galactic skycut made by the WMAP team. In order to reduce possible residual contamination from the Galactic foreground, we further cut a region $|b| < 35^\circ$ for the Q+V+W map. In order to estimate the errors, firstly, we generated 1000 random positions on the ILC($|b| > 20^\circ$) and the Q+V+W($|b| > 35^\circ$) maps, and then computed variances σ^2 for the filtered mean temperature. Second, we have calculated expected σ^2 for the filtered temperature using the angular power spectrum C_l for the WMAP 7-year data obtained by the WMAP team (see appendix A). Note that we have computed pseudo- C_l 's from the C_l ' for each skymap. As shown in figure 11, the observed standard deviations are $1\sigma = 19 \sim 20 \mu\text{K}$ for $\theta_{in} = 4^\circ$ and $1\sigma = 14 \sim 16 \mu\text{K}$ for $\theta_{in} = 14^\circ$. Our result for $\theta_{in} = 4^\circ$ is roughly consistent with the values for stacked images in Granett et al. (2008) assuming no correlation between voids/clusters in a particular configuration. In the Q+V+W map, a slight suppression in σ is observed at $\theta_{in} > 8^\circ$. Theoretically calculated values are found to be systematically larger than the observed values by 4 – 16 per cent for $4^\circ < \theta_{in} < 14^\circ$. These discrepancies represent an uncertainty due to the Galactic foreground emission.

As one can see in figure 12, a deviation corresponding to the inner peak is roughly 4σ and that corresponding to the outer peak is $4 \sim 4.5\sigma$. Assuming that the filtered mean temperature obeys Gaussian statistics, the statistical significances are $P(> 4\sigma) = 6 \times 10^{-3}$ and $P(> 4.5\sigma) = 7 \times 10^{-4}$ per cent. The total solid angle of the ILC map ($|b| > 20^\circ$) is 8.27 sr and that of the Q+V+W map ($|b| > 35^\circ$) is 5.36 sr. The total number of the independent

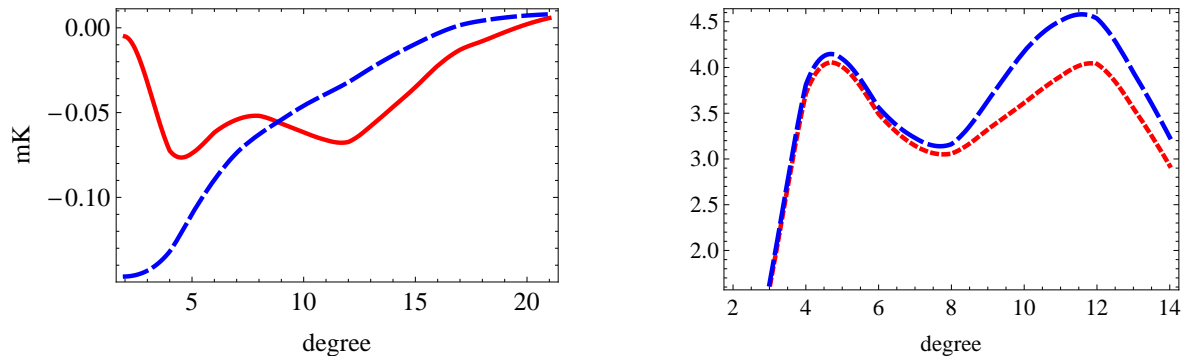


Fig. 12.— Left: mean temperature around the center of the cold spot for a compensating top-hat filter (full curve) and for an uncompensated top-hat filter without a ring (dashed curve) as a function of the inner radius θ_{in} . Right: mean temperature around the center of the cold spot divided by the standard deviation for the ILC map (dashed curve) and the Q+V+W map (full curve) as a function of θ_{in}

patches is roughly given by a ratio between the solid angle of the map and the area of the spherical patch with angular radius θ_{out} . Therefore, for $\theta_{out} = 7^\circ$, we have ~ 180 samples for the ILC and ~ 110 samples for Q+V+W map, yielding $(110 - 180) \times P(> 4\sigma) = 0.7 - 1$ per cent. In a similar manner, for $\theta_{out} = 17^\circ$, one can easily show that the statistical significance is $\sim 0.01 - 0.2$ per cent corresponding to $\sim 3\sigma$ if the likelihood function is a Gaussian one.

Thus, the cold spot surrounded by a hot ring at scale $\sim 17^\circ$ is more peculiar than the cold spot at scale $\sim 7^\circ$. Therefore, the real size of the supervoid is expected to be larger than the apparent size of the cold region. Because 1σ deviation (corresponding to $15 - 20\mu\text{K}$) due to a supervoid is enough to make the signal non-Gaussian, it is reasonable to assume that the contribution from a supervoid is less important than that from other effects due to acoustic oscillation or Doppler shift at the last scattering surface. For instance, a supervoid with a density contrast $\delta_m = -0.3$ with a comoving radius $r = 200 h^{-1}\text{Mpc}$ at a redshift $z = 0.2$ corresponding to an angular radius $\theta = 20^\circ$ would yield a temperature decrease $\Delta T \sim 20\mu\text{K}$ in the direction to the center. Moreover, if the supervoid is not compensating, a wall surrounding the supervoid could generate the observed hot ring. It may consist of just an ordinary underdense region surrounded by massive superclusters. Further observational study is necessary for checking the validity of the “local supervoid with a massive wall” scenario.

4. Possible solutions

Why the observed ISW signals for prominent structures are so large?

One possible systematic effect may come from a deviation from spherically symmetric density profile that we have not considered. Indeed, gravitational instability causes pancake or needle like structures in high density regions. However, as we have seen, the order of the density contrast of relevant prominent super-structures is $\delta_m = O[0.1]$. Therefore, we expect that the effect of anisotropic collapse plays just a minor role. Moreover, in the case of supervoids, a deviation from spherical symmetry is suppressed as the void expands in comoving coordinates. Thus, it is difficult to attribute the major cause to the deviation from spherical symmetry.

Another possible systematic effect is our neglect of fluctuations on larger scales. For instance, we may have observed just a tip of fluctuations whose real scale extends to $r > 1000 h^{-1}\text{Mpc}$. Indeed, the amplitude of the ISW effect is roughly proportional to the scale of fluctuations, i.e., $\Delta T/T \propto r$ for $r > 100 h^{-1}\text{Mpc}$ (Inoue & Silk 2007). Therefore, the observed large amplitude of ISW signal can be naturally explained. However, the angular sizes of the observed hot and cold spots from the stacked images are just $4^\circ - 6^\circ$ at $z \sim 0.5$ corresponding to $r = 100 - 140 h^{-1}\text{Mpc}$. It is difficult to explain why the angular sizes are so small since contributions from the ordinary Sachs-Wolfe effect and the early ISW effect generated near the last scattering surface are significantly suppressed by stacking a number of images.

Then what are the possible mechanisms that can explain the anomalously large ISW signals?

One intriguing possibility is that the primordial fluctuations are non-Gaussian. Our results suggest that the number of both supervoids and superclusters is significantly enhanced in comparison with the standard Gaussian predictions. Therefore, the effect of deviation from Gaussianity may appear in the statistics of 4-point correlations in real space or trispectrum in harmonic space. It can be also measured by the Minkowski functionals that contain information of 4-point or higher order correlations. At the last scattering surface, the comoving scale of $300 h^{-1}\text{Mpc}$ corresponds to angular scale $\sim 2^\circ$. If the background universe is homogeneous, such a non-Gaussian feature must appear at the CMB anisotropy at multiple $l \sim 100$ corresponding to angular scale $\sim 2^\circ$ as well. However, so far no such a noticeable deviation from Gaussianity in the CMB anisotropies has been observed (Vielva & Sanz 2010). Therefore, it is difficult to explain the observed signals by a simple non-Gaussian scenario unless one gives up the cosmological Copernican principle (Tomita 2001).

Another possibility is a certain feature on the power spectrum of primordial fluctuation

(Ichiki et al. 2009). Spike-like features in the primordial power spectrum appear in some inflationary scenarios that produce primordial black holes (Ivanov et al. 1994, Juan et al. 1996, Yokoyama 1998). Although there is no natural reason to have a feature only on the scale of super-structures ($l \sim 100$), observational constraints are not stringent since one needs to increase the number of samples if one abandons an assumption of the smoothness of the primordial power.

Some cosmological models containing time evolving dark energy/quintessence or those based on scalar-tensor gravity predict an enhancement in the ISW effect due to an enhancement in acceleration of the cosmological expansion or non-trivial time evolution of dark energy or scalar field that may couple to matter or metric (Amendola 2001, Nagata et al. 2003). This may help to explain the anomalously large ISW signal. However, at the same time, we need to suppress the ISW contribution on large angular scales since the observed angular power of the CMB anisotropy at very large angular scales $l \sim 2$ is relatively low. Models based on some alternative gravity might be helpful for realizing these observational features (Afshordi et al. 2009).

5. Conclusion

In this paper, we have shown that recent observations imply a presence of quasi-linear super-structures with a comoving radius $100 - 300 h^{-1}\text{Mpc}$ at redshifts $z < 1$. Observations are at odd with the concordant ΛCDM cosmology that predicts Gaussian primordial perturbations at $> 3\sigma$ level.

First, we have developed a formalism to estimate the amplitude of the ISW signal for prominent structures based on thin-shell approximation and the homogeneous collapse model. From comparison with other calculations based on perturbation theory and the LTB solution, we have found that our simple model works well for estimating the ISW signal for quasi-linear superstructures in Λ -dominated universes.

Secondly, we have applied our developed tools to observations of the ISW signals using the SDSS-LRG catalog, the 2MASS catalog, and the cold spot in the Galactic southern hemisphere in the WMAP data. The ISW signals from stacked images for the SDSS-LRG catalog is inconsistent with the predicted values in the concordant ΛCDM model at more than 3σ . The radial profiles of the stacked image show a hot-ring around a cold spot for voids and a dip at the center of a hot-spot without a cold-ring for clusters. These non-linear features are also reproduced by our models using the LTB solutions although the agreement is not perfect. The asymmetrical features suggest that the observed super-structures are in

quasi-linear regime rather than linear regime. The amplitudes of the ISW signals obtained from the 2MASS catalog at redshifts $0.1 < z < 0.3$ are found to be several times larger than expected values. We have confirmed that the mean temperature around the cold spot filtered by a compensating top-hat filter with angular scale $\theta_{out} = 16^\circ - 17^\circ$ deviates from the mean value at roughly 3σ level suggesting a presence of a hot-ring around the cold spot. Note that our finding is consistent with the previous result that the cold spot itself is not unusual but the hot-ring plus the inner cold region is found to be unusual (Zhang & Huterer 2009). This implies that a supervoid may reside at low redshift $z < 0.3$ and the angular size may be larger ($= 16^\circ - 17^\circ$) than considered in literature (Masina & Notari 2009).

Finally, we have discussed possible causes of the discrepancy between the theory and observation, namely, observational systematics, primordial non-Gaussianity, features in power spectrum, dark energy or alternative gravity.

We have not considered effects of non-spherical collapse which are important for improving estimation of the mass function of non-linear objects and effects of uncompensated mass distribution for super-structures. The extension of our analysis to more elaborate ones incorporating these effects would be helpful for realizing detailed comparison between the theory and the observation.

Future surveys of the CMB, galaxy distribution, weak lensing and theoretical studies on dark energy/alternative gravity and inhomogeneous cosmology will certainly yield fruitful results for solving the puzzle.

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A. First-order and second-order ISW effects

In what follows, we derive analytic formulae for computing temperature fluctuations due to the ISW effect for spherically symmetric compensated top-hat type density perturbations using first-order and second-order perturbation theory (Tomita & Inoue 2008, abbreviated as TI hereafter). The relation between density perturbations of the growing mode and the potential function F of spatial variables are given in equations (3.6) and (3.11) of TI. The top-hat type density profile is parametrized in terms of two constants b and c , representing first order density contrasts at the center and at the wall at the present time (figure A1).

The first-order density contrast δ_m^L at a conformal time η when the scale factor is equal to $a(\eta)$ can be written in terms of the background matter density ρ_B and the growth function $P(\eta)$ corresponding to the growing mode of density perturbation as

$$\delta_m^L \equiv \delta\rho^{(1)}/\rho_B = \frac{1}{\rho_B a^2} [(a'/a)P' - 1](c, -b), \quad (\text{A1})$$

for $(r < r_0, r_0 < r < r_1)$, respectively, where a prime denotes a partial derivative with respect to conformal time η .

The second-order density contrast is expressed as

$$\delta_m^S \equiv \delta\rho^{(2)}/\rho_B = \frac{2/3}{\rho_B a^2} \left\{ (\zeta_1 - \frac{9}{2}\zeta_2)c^2, [2(c+b)^2(r_0/r)^6 + b^2]\zeta_1 + \frac{9}{2}[(c+b)^2(r_0/r)^6 - b^2]\zeta_2 \right\} \quad (\text{A2})$$

for $(r < r_0, r_0 < r < r_1)$, respectively, where ζ_1 and ζ_2 are given in equation (2.19) of TI. Here we have omitted the terms that are negligible if $r_1 \ll H^{-1}$ because we assume that typical size of super-structures is $O(100)h^{-1}\text{Mpc}$. Neglecting the terms higher than second-order, the total density contrast $\delta\rho/\rho_B$ can be written as

$$\frac{\rho_B}{\rho_B + \delta\rho} = \frac{1}{1 + \delta\rho/\rho_B} = 1 - \delta_m^L + (\delta_m^L)^2 - \delta_m^S. \quad (\text{A3})$$

For a central value of total density contrast, $(\delta\rho/\rho)_c$, we have the relation

$$\alpha(z)c^2 + \beta(z)c - \gamma(z) = 0, \quad (\text{A4})$$

where z is the redshift, $\alpha(z) \equiv \frac{2}{3}(\zeta_1 - \frac{9}{2}\zeta_2)/(\rho_B a^2) - [\beta(z)]^2$, $\beta(z) \equiv (a'p' - 1)/(\rho_B a^2)$ and $\gamma(z) \equiv 1 - 1/[1 + (\delta\rho/\rho)_c]$.

In the text we consider models of supervoids and superclusters with a given set of r_0, r_1 and $(\delta\rho/\rho)_c(z_c)$, where $(\delta\rho/\rho)_c(z_c)$ is $(\delta\rho/\rho)_c$ at the epoch of redshift z_c . From this set we obtain c and b , solving the above equation as

$$c = -[\beta(z_c) + \sqrt{\beta(z_c)^2 + 4\alpha(z_c)\gamma(z_c)}]/(2\alpha(z_c)), \quad (\text{A5})$$

and b is related to c as $b/c = 1/[(r_1/r_0)^3 - 1]$ for compensated super-structures.

The first-order and second-order temperature fluctuations $\Delta T^{(1)}/T$ and $\Delta T^{(2)}/T$ are defined by equations (4.2) and (4.4) of TI. Their expressions for a light path passing the center of spherical voids and clusters are given in equations (5.11) and (5.13) of TI. For the other light paths, the first-order temperature fluctuation is derived from equations (5.8) and (5.9) with equation (C6) of TI and expressed as

$$\Delta T^{(1)}/T = -2\left\{\frac{a'}{a} + \left[\frac{a''}{a} - 3\left(\frac{a'}{a}\right)^2\right]P'\right\}c(r_0)^3 J(r/r_0), \quad (\text{A6})$$

where $J(r/r_0)$ is given in equation (C7) of TI for $r \leq r_0$. For $r_1 > r > r_0$, we have

$$J(u) = -\frac{1}{6}2u_1^2[I_1 + I_3 - 3u_1^2 I_2]/(u_1^3 - 1), \quad (\text{A7})$$

where $u \equiv r/r_0$, $u_1 \equiv r_1/r_0$ and

$$\begin{aligned} I_1 &= \int_u^{u_1} (u^2 - u^2)^{-1/2} du = \ln[(u_1 + \sqrt{u_1^2 - u^2})/u], \\ I_2 &= \int_u^{u_1} (u^2 - u^2)^{-1/2} u du = (u_1^2 - u^2)^{1/2}, \\ I_3 &= \int_u^{u_1} (u^2 - u^2)^{-1/2} u^3 du = \frac{1}{3}(u_1^2 + 2u^2)(u_1^2 - u^2)^{1/2}. \end{aligned} \quad (\text{A8})$$

The second-order temperature fluctuation can be derived from equations (2.17),(2.18),(4.4) and (4.5) of TI and expressed as

$$\Delta T^{(2)}/T = -[\zeta'_1 \int_0^\infty d\lambda (F_{,r})^2 + \zeta'_2 \times 100 \int_0^\infty d\lambda \Phi_0], \quad (\text{A9})$$

where $\int_0^\infty d\lambda (F_{,r})^2$ and $\int_0^\infty d\lambda \Phi_0$ for $r < r_0$ are given in equations (C3) and (C4) of TI and the expression of ζ'_1 and ζ'_2 is shown in equations (4.6) and (4.7) of TI. For $r_1 > r > r_0$, we have

$$\begin{aligned} \int_0^\infty (F_{,r})^2 d\lambda &= \frac{1}{9}c^2(r_0)^3[u_1^6 I_4 + I_3 - 2u_1^3 I_1]/(u_1^3 - 1)^2 \\ 100 \int_0^\infty \Phi_0 d\lambda &= \frac{1}{4}c^2(r_0)^3[u_1^6 I_4 - 2I_3 + 9u_1^2 I_2 - 8u_1^3 I_1]/(u_1^3 - 1)^2, \end{aligned} \quad (\text{A10})$$

where I_i ($i = 1 - 3$) are given above and I_4 is

$$I_4 = \int_u^{u_1} (u^2 - u^2)^{-1/2} u^{-3} du = \frac{1}{2u^2} \left\{ \frac{1}{u} \tan^{-1}[\sqrt{u_1^2 - u^2}/u] + \sqrt{u_1^2 - u^2}/u^2 \right\}. \quad (\text{A11})$$

When we compare the temperature fluctuations in the perturbative model (in appendix A) and those in the full non-linear model (in appendix B), we should notice the difference

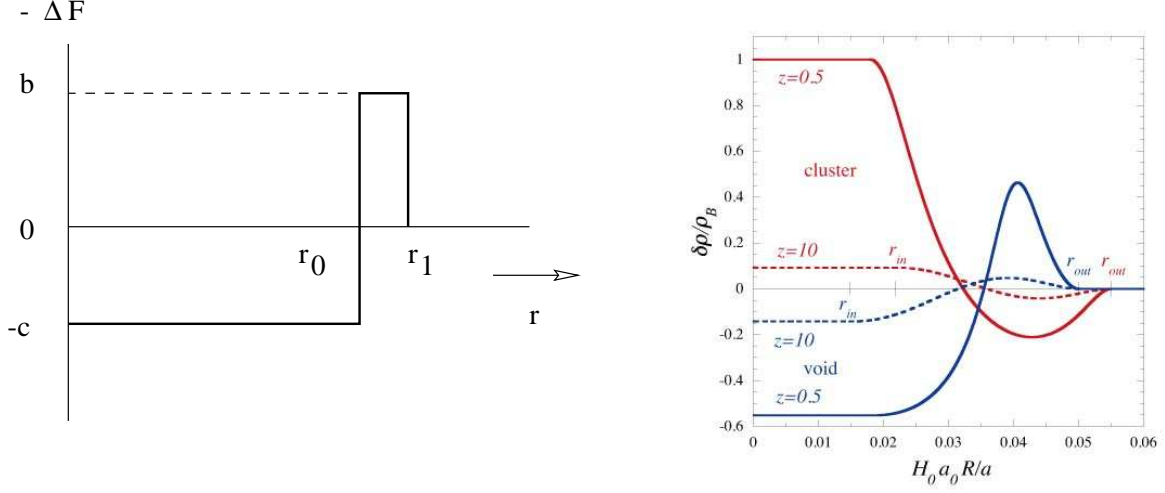


Fig. 13.— (Appendix A,B) Left: matter density contrast (at linear order) for a top-hat type spherical void used in our perturbative analysis. Right: density profiles δ_m in our LTB models as a function of physical radius in unit of present Hubble radius for a compensated cluster and void. The initial condition is set at a redshift $z = 10$. We assume a concordant FRW cosmology with $(\Omega_{m,0}, \Omega_{\Lambda,0}) = (0.26, 0.74)$ as the background spacetime.

of their density profiles, i.e. the top-hat profile (in appendix A) and the Sakai-Inoue (SI) profile (in appendix B). For our comparison in this paper, we simulate the top-hat profile to the SI profile by equating the outer boundaries and their zero points as follows. Here we represent the SI profile using the radial coordinate r defined in the perturbative model. In the top-hat profile, the radii in the outer boundary and the zero point are $r = r_1$ and r_0 , respectively, and in the SI profile the radius in the outer boundary is $r = r_{out}$ and the zero point is $r = (r_{in} + r_c)/2$ approximately, in which $r_c = (r_{out} + r_{in})/2$. If we equate these outer boundaries and zero points, we obtain

$$r_0 = (r_{in} + r_c)/2 = (3r_{in} + r_{out})/4. \quad (\text{A12})$$

Then for relative widths $w_{th} \equiv 1 - r_0/r_1$ and $w_{SI} \equiv 1 - r_{in}/r_{out}$, we have a relation $w_{th} = \frac{3}{4}w_{SI}$. In the text we show the temperature fluctuations in both models with parameters which satisfy this relation.

B. Method of computing Rees-Sciama effects using the LTB models

Any spherically symmetric spacetime which includes dust of energy density $\rho(t, r)$ and a cosmological constant Λ can be described by the LTB solution,

$$ds^2 = -dt^2 + \frac{R'^2(t, r)}{1 + f(r)} dr^2 + R^2(t, r)(d\theta^2 + \sin^2 \theta d\varphi^2) \quad (\text{B1})$$

which satisfies

$$\dot{R}^2 = \frac{2Gm(r)}{R} + \frac{\Lambda}{3}R^2 + f(r) \quad (\text{B2})$$

$$\rho = \frac{m'(r)}{4\pi R^2 R'} \quad (\text{B3})$$

where $' \equiv \partial/\partial r$ and $\dot{} \equiv \partial/\partial t$.

Our model is composed of three regions: an outer flat FRW spacetime, an inner negatively/positively curved FRW spacetime, and an intermediate shell region described by the LTB metric. At the initial time $t = t_i$, which we choose as $z_i = 10$, we define the radial coordinate as $R(t_i, r) = r$, and we assume (figure A1)

$$m(r) = \frac{4}{3}\pi R^3 \rho_m W(r), \quad W(r) \equiv \begin{cases} 1 + \delta_m & \text{for } r \leq r_{in} \\ 1 + \frac{\delta_m}{16}(8 - 15X + 10X^3 - 3X^5) & \text{for } r_{in} \leq r \leq r_{out} \\ 1 & \text{for } r \geq r_{out} \end{cases} \quad (\text{B4})$$

where

$$X \equiv \frac{r - r_c}{w/2}, \quad r_c \equiv \frac{r_{out} + r_{in}}{2}, \quad w \equiv r_{out} - r_{in}. \quad (\text{B5})$$

Initial velocity field, $v = \dot{R} - HR$, is given by the linear perturbation theory (TI). Then $f(r)$ is determined by equation (B2). Our model parameters are $\Omega_{m,0}$, r_{out} , w , the redshift of the center of a void/cluster, z_c , and $\delta_m(z_c)$.

The wave 4-vector k^μ of a photon satisfies the null geodesic equations,

$$k^\mu \equiv \frac{dx^\mu}{d\lambda}, \quad k^\mu k_\mu = 0, \quad \frac{dk^\mu}{d\lambda} + \Gamma_{\nu\sigma}^\mu k^\nu k^\sigma = 0 \quad (\text{B6})$$

For null trajectories on the $\theta = \pi/2$ plane, the geodesic equations (B6) with the metric (B1) reduce to

$$(k^t)^2 = X^2 + R^2(k^\varphi)^2, \quad X \equiv \frac{R'}{\sqrt{1+f}} k^r, \quad (\text{B7})$$

$$R^2 k^\varphi = \text{constant}, \quad (\text{B8})$$

$$\frac{dr}{dt} = \frac{k^r}{k^t}, \quad \frac{d\varphi}{dt} = \frac{k^\varphi}{k^t}, \quad (\text{B9})$$

$$\frac{dk^t}{dt} = -\frac{\dot{R}'X^2}{R'k^t} - R\dot{R}\frac{(k^\varphi)^2}{k^t}, \quad (\text{B10})$$

$$\frac{dX}{dt} = -\frac{\dot{R}'}{R'}X + R\sqrt{1+f}\frac{(k^\varphi)^2}{k^t}. \quad (\text{B11})$$

We use the null condition (B7) not only to set up initial data but also to check numerical precision after time-integration.

To integrate the geodesic equations (B9)-(B11) together with the field equations (B2) and (B13) numerically, we discretize r into N elements,

$$r_i = i\Delta r, \quad i = 1, \dots, N, \quad \Delta r = \frac{r_{out}}{N}. \quad (\text{B12})$$

and any field variable $\Phi(t, r)$ into $\Phi_i(t) \equiv \Phi(t, r_i)$. Evolution of $R_i(t)$ is determined by (B2), but we also need data of $R'_i(t)$ and $R''_i(t)$. Because finite difference approximation, $R'_i(t) \approx (R_{i+1} - R_{i-1})/(2\Delta r)$, include errors of $O(\Delta r^2)$, we numerically integrate with time,

$$\dot{R}' = \frac{1}{2\dot{R}} \left(\frac{2Gm'}{R} - \frac{2Gm}{R^2}R' + f' + \frac{2}{3}\Lambda RR' \right), \quad (\text{B13})$$

which is given by differentiating (B2) with respect to r . Furthermore, to vanish $R''(t, r)$ in the geodesic equations, we have introduced an auxiliary variable X . To prepare geometrical values between grid points r_i and r_{i+1} , we adopt cubic interpolation: at each time t_* any variable $\Phi(t_*, r)$ in $r_i < r < r_{i+1}$ is determined by

$$\begin{aligned} \Phi(t_*, r) &= ax^3 + bx^2 + cx + d, \quad x \equiv r - r_i - \frac{\Delta r}{2} \\ a &\equiv \frac{-\Phi_{i-1} + 3(\Phi_i - \Phi_{i+1}) + \Phi_{i+2}}{6(\Delta r)^3}, \quad b \equiv \frac{\Phi_{i-1} - \Phi_i - \Phi_{i+1} + \Phi_{i+2}}{4(\Delta r)^2}, \\ c &\equiv \frac{\Phi_{i-1} + 27(-\Phi_i + \Phi_{i+1}) - \Phi_{i+2}}{24\Delta r}, \quad d \equiv \frac{-\Phi_{i-1} + 9(\Phi_i + \Phi_{i+1}) - \Phi_{i+2}}{16}, \end{aligned} \quad (\text{B14})$$

We also have to consider null geodesics from an observer to the void/cluster. Suppose that the observer is at the origin and the center of the void/cluster is located at $x = x_c$ on the y axis. Then, without loss of generality, on the x - y plane we can analyze light rays which reach the observer. Some position on the outer shell and the four momentum of the light there in the observer-centered coordinate are given by

$$x = x_c + r_{out} \cos \varphi, \quad y = r_{out} \sin \varphi, \quad (\text{B15})$$

$$k_{(O)}^\mu = E \left(1, -\frac{\cos \alpha}{a}, -\frac{\sin \alpha}{a}, 0 \right), \quad (\text{B16})$$

where E is the photon energy and α is the angle between the light ray and the x -axis. Defining $l(z)$ as a comoving length from the observer to the photon, we can write the light path as

$$x = l(z) \cos \alpha, \quad y = l(z) \sin \alpha, \quad l(z) = \int_0^z \frac{dz}{a_0 H(z)}. \quad (\text{B17})$$

The solution of (B15) and (B17) gives

$$l = x_c \cos \alpha - \sqrt{r_{out}^2 - x_c^2 \sin^2 \alpha}, \quad \sin \varphi = \frac{l}{r_{out}} \sin \alpha, \quad (\text{B18})$$

and the null vector in the void/cluster-centered spherical coordinate,

$$k^t = E, \quad k^r = -\frac{E \cos(\alpha - \varphi)}{r_{out}}, \quad k^\varphi = -\frac{E \sin(\alpha - \varphi)}{a r_{out}}. \quad (\text{B19})$$

at the time when the photon leaves the shell, z_{leave} .

Our computing algorithm is summarized as follows:

- (i) Suppose $\Omega_{m,0}$, $\delta_m(z_c)$, r_{out} , w and redshift of the center of a void/cluster, z_c . For each angle, α , Eqs.(B18) and (B19) give “initial” conditions of the null geodesic at $z = z_{\text{leave}}$ in the void/cluster-centered spherical coordinate.
- (ii) Solve the field equations in the LTB spacetime, (B2) and (B13), from $z = z_i$ to z_{leave} .
- (iii) Solve the null geodesic equations (B7)-(B11) together with the the field equations (B2) and (B13) backward from $z = z_{\text{leave}}$ to the time when the photon enters the shell.

C. Temperature Variance for top-hat compensating filter

In what follows, we derive analytic formulae for computing variance of temperature fluctuations on a sky for a circular top-hat compensating filter $W_{th}(\theta; \theta_{in})$. We assume that an ensemble of the CMB fluctuations can be regarded as an isotropic random field on unit sphere S^2 . Let $\Delta T(\theta, \phi)$ be a temperature fluctuation at spherical coordinates (θ, ϕ) . Then filtered temperature fluctuation centered at the “north” pole ($\theta = 0$) can be written as

$$\Delta T_f = A^{-1} \int_0^{2\pi} d\phi \int_0^{\theta_{out}} d\theta \sin \theta W_{th}(\theta; \theta_{in}) \Delta T(\theta, \phi), \quad (\text{C1})$$

where $A = 2\pi(1 - \cos \theta_{in})$. Plugging ΔT expanded in spherical harmonics Y_{lm} ,

$$\Delta T = \sum_{lm} a_{lm} Y_{lm} \quad (C2)$$

into equation (A1), we have

$$\Delta T_f = A^{-1} \sum_l a_{l0} G_l, \quad (C3)$$

where

$$\begin{aligned} G_l &= \frac{\sqrt{\pi(2l+1)}}{l+1} \left[2 \left(-x_{in} P_l(x_{in}) + P_{l-1}(x_{in}) \right) + x_{out} P_l(x_{out}) - P_{l-1}(x_{out}) \right], \\ x_{in} &= \cos \theta_{in}, \quad x_{out} = 2x_{in} - 1. \end{aligned} \quad (C4)$$

Note that we have used a formula for the Legendre function P_l ,

$$\frac{dP_l(x)}{dx} = \frac{l(l+1)}{1-x^2} \int_x^1 P_l(x) dx \quad (C5)$$

in deriving equation A4. Because ΔT is assumed to be isotropic on S^2 , the variance of ΔT_f can be written as a function of the angular power spectrum C_l as

$$\sigma_f^2 = A^{-2} \sum_l C_l G_l^2. \quad (C6)$$

If the CMB sky is smoothed by a Gaussian beam with the FWHM θ_s , then the variance is

$$\sigma_f^2 = A^{-2} \sum_l C_l B_l G_l^2, \quad (C7)$$

where $B_l = \exp[-\sigma_s^2 l(l+1)/2]$, and $\sigma_s = (8 \ln 2)^{-1/2} \theta_s$.

In the absence of complete sky coverage, we cannot directly observe C_l . We can only compute estimated expansion coefficients for the observed region R in the sky (Bunn 1995),

$$b_{lm} = N_{lm} \int_R \Delta T Y_{lm} \sin \theta d\theta d\phi, \quad (C8)$$

where N_{lm} is a factor chosen to normalize b_{lm} appropriately. If R is azimuthally symmetric, one possible prescription is to set (Peebles, 1973)

$$N_{lm} = W_{lm}^{-1/2}, \quad (C9)$$

where

$$W_{lm} = \int_R Y_{lm} Y_{l'm} \sin \theta d\theta d\phi. \quad (C10)$$

Then a possible estimator for C_l is given by

$$\tilde{C}_l \equiv \langle |b_{lm}|^2 \rangle = \frac{1}{2l+1} \sum_{l',m} W_{llm}^{-2} W_{l'm}^2 C_{l'}. \quad (\text{C11})$$

In the limit that C_l varies much more slowly than $W_{l'm}$

$$\frac{1}{C_l} \frac{dC_l}{dl} \ll \frac{1}{W_{l'm}} \left. \frac{dW_{l'm}}{dl'} \right|_{l=l'}, \quad (\text{C12})$$

we have $\tilde{C}_l \approx C_l$.

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